

# A combination of regression and internal point methods as a hybrid model for estimating oat plant productivity

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**ABSTRACT.** The internal points method (IPM-Carvalho), with regression analysis, can generate an efficient hybrid model for estimating oat grain productivity. We tested a combination of the internal points method and regression to estimate straw productivity. We also applied this methodology to forecast a harvest index in the elaboration of a hybrid model to estimate oat grain productivity, taking into account nitrogen management and growth regulator use, with biological and environmental indicators. Simulation of oat yield as a function of nitrogen and growth regulator applications, with biological and environmental inputs, can assist in the development of more efficient and sustainable management for this crop. Two experiments were conducted during 2013, 2014, and 2015; one was used to quantify biomass yield and the other to determine grain yield and plant lodging. The experimental design was randomized blocks with four replications in a 4 x 3 factorial scheme in the sources of variation, which were growth regulator (0, 200, 400 and 600 mL ha<sup>-1</sup>) and nitrogen (30, 90 and 150 kg ha applications. The environmental parameters that were included were rainfall and maximum air temperature. The nitrogen was applied as urea at the expanded fourth leaf stage. The growth regulator was trinexapac-ethyl applied at the

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stage between the 1st and 2nd visible stem node. Straw productivity was obtained by the IPM model with nitrogen dose and rainfall inputs. The harvest index was obtained by regression as a function of the growth regulator doses. The combination of the internal points method to estimate straw productivity with the use of regression in the forecast of the harvest index proved to be a useful model for estimating oat grain productivity based on biological and environmental parameters, together with nitrogen and growth regulator applications.

**Key words:** Avena sativa; Rainfall; Nitrogen; Growth regulator; IPM-Carvalho; Sustainability

#### INTRODUCTION

Oats are considered a multi-purpose cereal, used for soil cover due to the high volume of straw it produces (Godoy et al., 2016; Queiroz et al., 2017), and for animal feed in the form of pasture, hay, silage, and as an ingredient of feed concentrates (Romitti et al., 2017; Dornelles et al., 2018). For human consumption, it is possible to produce numerous products, with nutritional and functional qualities superior to those of other cereals (Sancho and Pastore, 2016; Mantai et al., 2017). Therefore, the high demand for grain and derivatives has resulted in an increase in the crop area, as well as increases in yield through the use of new technologies (Arenhardt et al., 2017; Scremin et al., 2017).

High yield of oats depends on the genetic potential of the cultivars, management technologies, and favorable climate and soil for growing (Hawerroth et al., 2015; Krysczun et al., 2017). Among the management technologies, nitrogen fertilization stands out, as it is the nutrient most absorbed by cereals and most directly linked to yield. The need to supply this nutrient via synthetic fertilizers is highlighted due to the insufficient quantity released by the soil during the growing cycle (Brezolin, et al., 2017; Costa et al., 2018; Veçozzi et al., 2018). However, although increases in nitrogen doses, along with favorable weather conditions, favor increased grain yield, they also promote vegetative growth, facilitating lodging (Kaspary et al., 2015; Mantai et al., 2017).

Lodging is the phenomenon in which the plant loses its vertical orientation, leans and falls to the ground, resulting in curved plants or even stem breaking. Lodging leads to losses in grain yield and quality and difficulties at harvest (Chavarria et al., 2015; Krysczun et al., 2017). In an effort to minimize lodging, the use of growth regulators in crops such as crotalaria (Kappes et al., 2011; Araújo et al., 2018), rice (Alvarez et al., 2014; Goes et al., 2015), wheat (Schwerz et al., 2015; Ferreira et al., 2017) and oats (Guerreiro and Oliveira, 2012; Hawerroth et al., 2015; Marolli, et al., 2017b) has been studied. These regulators are chemical compounds that reduce the length of the inter-nodes by obstructing the biosynthesis of gibberellic acid, making the plant smaller in size and thickening the stem, favoring yield with significant reduction or even absence of lodging (Kaspary et al., 2015; Fagherazzi et al., 2018).

The development of models for the simulation and optimization of processes involving linear and non-linear effects of agroecosystems has increased (Klering et al., 2016; Marolli et al., 2017a). Although there are models for simulating the yield of grains in

cereals, they do not simultaneously include management, biological and environmental indicators that are decisive for yield (Souza et al., 2013; Rosa et al., 2015). Models that represent reality through one or more equations and algorithms enable efficient and low-cost simulations (Corrêa et al., 2011; Mamann et al., 2017). Along this line, Carvalho et al. (2009) developed an internal parameters model (IPM) for the calculation of dry oat matter. Computational limitations until a few years ago meant that models with biological significance and statistical analyses proved to be conflicting, leading to a preference for linear and/or polynomial models instead of non-linear models, provider greater biological significance. Computational advances currently allow the use of nonlinear models with regression, enabling the generation of more efficient hybrid models.

The IPM-Carvalho model simulates straw yield involving rainfall and nitrogen supply (Carvalho et al., 2009; Borges et al., 2012). The regression model allows estimation of the ideal input with simulation of the harvest index. The harvest index represents the relationship between grain yield and biological yield (straw + grains), determining the efficiency with which photoassimilates are converted into straw and grain (Silva et al., 2011; Silva et al., 2015).

An alternative that can be implemented is the construction of a hybrid model associating the internal points method (IPM-Carvalho) with a regression model, simultaneously including biological, environmental and nitrogen management indicators and growth regulator applications. This possibility can assist in the development of more efficient and sustainable processes and applications for simulation of grain productivity on mobile devices, facilitating the forecast of harvest in surveys of agricultural activity guarantee programs. Along this line, we examined whether the internal points method (IPM-Carvalho) to estimate straw productivity together with regression to forecast the harvest index could be used to develop a hybrid model that would efficiently estimate oat productivity, involving nitrogen management and growth regulator applications together with biological and environmental indicators.

### MATERIAL AND METHODS

The experiments were conducted in the field in the 2013, 2014 and 2015 crop seasons in Augusto Pestana, RS, Brazil (28°26'30'' S latitude and 54°00'58'' W longitude). The soil of the experimental area is classified as a Typical Dystrophic Red Latosol (Oxisol) and the climate according to the classification of Köppen, of the Cfa type, with hot summer and without a dry season. Ten days before sowing, soil analysis was carried out, identifying the following chemical characteristics (Tedesco et al., 1995): pH = 6.2, P = 33.9 mg dm<sup>-3</sup>, K = 200 mg dm<sup>-3</sup>, OM = 3.0%, Al = 0 cmolc dm<sup>-3</sup>, Ca = 6.5 cmolc dm<sup>-3</sup> and Mg = 2.5 cmolc dm<sup>-3</sup>. Sowing was carried out with a seeder-fertilizer in a soybean/oat system. At sowing, 30 and 20 kg ha<sup>-1</sup> of P<sub>2</sub>O<sub>5</sub> and K<sub>2</sub>O were applied, respectively, based on the levels of P and K present in the soil, for the expectation of grain yields of 3 t ha<sup>-1</sup> and 10 kg ha<sup>-1</sup> of N at sowing, with the remainder to contemplate the doses indicated in the study, applied at the expanded fourth leaf stage (V4), with the source urea. The seeds were submitted to a germination and vigor test in the laboratory in order to correct the density for 400 seeds m<sup>-2</sup>. During the study, two applications of the tebuconazole fungicide were applied at a 0.75 L ha<sup>-1</sup> and weed control with metsulfuron-methyl herbicide at 4 g ha<sup>-1</sup>. The growth regulator

(Trinexapac-Ethyl) was applied at the stage between the 1<sup>st</sup> and 2<sup>nd</sup> visible stem node, with a backpack sprayer at a pressure of 30 lb in<sup>-2</sup> by compressed CO<sub>2</sub>, with flat fan tips.

Two experiments were conducted, one to quantify the total biomass yield and the other to estimate grain yield and lodging. In both experiments, the experimental design was a randomized block with four replications, following a 4 x 3 factorial scheme, in the sources of variation doses of growth regulator (0, 200, 400, 600 mL ha<sup>-1</sup>) and nitrogen doses (30, 90, 150 kg ha<sup>-1</sup>), respectively, totaling 96 experimental units. Each experimental unit consisted of 5 lines 5 meters long and spaced 0.20 m apart, forming plots of 5 m<sup>2</sup>. The harvesting of the experiments to estimate the biomass and grain yield occurred manually by cutting the three central lines of each plot, a stage close to the harvest point (120 days), with grain moisture around 18%. The plots for grain harvesting were tracked with a stationary harvester and sent to the laboratory to correct grain moisture to 13%, after weighing and estimating grain yield, converted to the area of one hectare (GY, kg ha<sup>-1</sup>). The plots for analysis of biomass yield were directed to forced air oven at a temperature of 65 °C, until reaching constant mass for weighing and estimate of biomass yield, converted to the area of one hectare (BY, kg ha<sup>-1</sup>). From these determinations, the straw yield (SY, kg ha<sup>-1</sup>) was simulated by the subtraction BY-GY. The rainfall was obtained by the meteorological station located 200 m from the experiments. The values of the general average of yield together with the information of temperature and pluviometric precipitation were used to classify the agricultural years as favorable, intermediate, and unfavorable for growth.

For the development of the simulation models, it was necessary to implement and validate the IPM-Carvalho model to simulate straw yield. This model is characterized by a non-linear function, with upper and lower limiters of w and n, mathematically expressed by:

$$p(w, n) = aw^2 + bwn + cn^2 + dw + en + f$$
, with a and  $c \le 0$  (Eq. 1)

The central path type IPM model generates the  $\mu$ , barrier parameter used to originate a problem with equality restrictions and set at  $\mu$ > 0, incorporating restrictions and generating a two-dimensional box of the objective function through a "logarithmic barrier", solving an unrestricted nonlinear programming problem. Thus, the maximization of the  $\emptyset \mu$  (w, n) model is now presented as follows.

$$\emptyset_{\mu}(w, n) = p(w, n) + \mu B(w, n)$$
 (Eq. 2)

in which, B(w, n) is the logarithmic barrier of the model, obtained by the following function:

$$B(w,n) = \log(w_{II} - w) + \log(w - w_{I}) + \log(n_{II} - n) + \log(n - n_{I})$$
 (Eq. 3)

For the process to be satisfactory, a decrease of  $\mu$  is made until reaching the established criterion. The use of the logarithmic barrier allows the procedure to generate interior points, away from the boundary of the two-dimensional box of the restrictions. That is, for each  $\mu$ , the maximization of  $\emptyset \mu$  is achieved at an interior point in the set of viable solutions to the problem, when  $\mu$  tends to zero, the logarithmic barrier moves the point up close to the optimal solution. The  $\emptyset \mu$  (w, n) maximization is performed, to fixed  $\mu$ , and as

 $\emptyset\mu$  is a strictly concave function, by the first order conditions  $(w, n) = [w (\mu), n (\mu)]$  is defined as an optimal solution to the unrestricted nonlinear programming problem if:

$$\frac{\partial \phi_{\mu}(w,n)}{\partial w} = \frac{\partial p(w,n)}{\partial w} - \frac{\mu}{w_{\mu} - w} + \frac{\mu}{w - w_{l}} = 2aw + bn + d - \frac{\mu}{w_{\mu} - w} + \frac{\mu}{w - w_{l}} = 0 \tag{Eq. 4}$$

$$\frac{\partial \phi_{\mu}(w,n)}{\partial n} = \frac{\partial p(w,n)}{\partial n} - \frac{\mu}{n_u - n} + \frac{\mu}{n - n_l} = 2cn + bw + e - \frac{\mu}{n_u - n} + \frac{\mu}{n - n_l} = 0 \tag{Eq. 5}$$

By 
$$s_u = \frac{\mu}{w_u - w} > 0$$
;  $s_l = \frac{\mu}{w - w_l} > 0$ ;  $z_u = \frac{\mu}{n_u - n} > 0$ ;  $z_l = \frac{\mu}{n - n_l} > 0$ , the system (1) -

(2) can be written as a non-linear system, that is:

$$2aw + bn + s_l - s_u = -d (Eq. 6)$$

$$2cn + bw + z_l - z_u = -e (Eq. 7)$$

$$s_{u}(w_{u} - w) = u \tag{Eq. 8}$$

$$s_1(w - w_1) = u (Eq. 9)$$

$$z_{u}(n_{u} - n) = u$$
 (Eq. 10)

$$z_1(n - n_1) = u$$
 (Eq. 11)

By subtracting equation 11 from equation 6, the points close to the optimal solution are obtained, close to the central path, obtaining the yield value. By subtracting equation (11) from (8) the conditions of "approximate complementary clearances" are obtained. To test the model's viability, (6) is decreased from (7), representing the constraints of the dual problem. The advantages of dual solutions are innumerable, among them the possibility of providing economic information on the use of resources stands out, assisting in decision-making and crop planning (Tsuchiya and Oliveira, 2017). In this case, the variables  $s_u$ ,  $s_l$ ,  $z_u$ ,  $z_l$  represent the rate of change in yield, caused by the variation of the limits: rainfall and nitrogen doses.

The implementation of the numerical procedure for maximizing yield considers the parameter  $\mu > 0$  as a point close to  $[w(\mu), n(\mu), s_u(\mu), s_l(\mu), z_u(\mu), z_l(\mu)]$ . Each resolution of the non-linear system (6) - (11) is considered as an iteration and its solution is possible using the Newton method. The  $\mu$  decreases, repeating the process until the predetermined stop condition is satisfied. The result of these iterations is the IPM-Carvalho model for simulating straw yield:

$$p(w, n) = 15.6w + 15.4n - 11.10^{-3}w^2 - 51.10^{-3}n^2$$
 (Eq. 12)

In which:

p(w, n) – oat straw yield (kg ha<sup>-1</sup>); w - 45% of rainfall (mm); n – nitrogen dose (kg ha<sup>-1</sup>). When meeting the assumptions of homogeneity and normality via Bartlett's tests, analysis of variance was performed to detect the main and interaction effects. Adjustment of the linear regression equation was performed to estimate the ideal growth regulator dose for the lodging of oat plants by increasing the growth regulator doses. As it is an equation that describes the linear behavior of lodging, the possibility of lodging plants of a maximum of 5% was considered, a value added to the parameter y of the equation, obtained by:

$$x = \left[\frac{y \pm b_0}{\pm b_1}\right] \tag{Eq. 133}$$

According to Romitti et al. (2016), the value of up to 10% lodging of oat plants does not bring significant losses in grain yield. Afterwards, adjustment of the regression equation of degree two was performed to estimate the harvest index (HI), equation (14), of oats as a function of the growth regulator doses in reduced condition (30 kg ha<sup>-1</sup>), high (90 kg ha<sup>-1</sup>) and very high (150 kg ha<sup>-1</sup>) nitrogen fertilization.

$$HI = a \pm bx \pm cx^2 \tag{Eq. 144}$$

where a, b and c are coefficients obtained by polynomial regression and x and  $x^2$  are the growth regulator doses.

Based on the assumption of efficiency of the IPM-Carvalho model for the simulation of oat straw yield, a regression analysis is developed, in order to add to the model the management of the growth regulator to estimate the adjusted dose to reduce the lodging of plants. Thus, the implemented IPM-Carvalho model is expressed by:

$$p(w, n, dr) = 15.6w + 15.4n - 0.42dr - 11.10^{-3}w^2 - 51.10^{-3}n^2 - 2.10^{-3}dr^2$$
 (Eq.15)

In which:

p(w, n, dr) – White oat straw yield (kg ha<sup>-1</sup>)

w - 45% of rainfall (mm);

n – Nitrogen dose (kg ha<sup>-1</sup>);

dr – Regulator dose (mL ha<sup>-1</sup>).

With the IPM-Carvalho model in place and knowing that straw yield p(w, n, dr) is the difference between biomass yield and grain yield [p(w, n, dr) = BY - GY], and grain yield is equal to the product of biomass yield with the harvest index (GY = BY \* HI), models for simulating the yield of biomass and oat grains are obtained by:

$$GY = BY * HI$$
 (Eq.

156)

$$p(w, n, dr) = BY - GY (Eq. 17)$$

Substituting (14) in (15), the biomass yield simulation model (18) is obtained.

$$p(w, n, dr) = BY - BY * HI$$
 (Eq. 18)

$$p(w, n, dr) = BY(1 - HI)$$
 (Eq. 19)

$$BY = \frac{p(w, n, dr)}{1 - HI}$$
 (Eq. 20)

24)

Replacing (20) in (17), the grain yield simulation model (24) is obtained.

$$p(w, n, dr) = \frac{p(w, n, dr)}{1 - HI} - GY$$

$$GY = \frac{p(w, n, dr)}{1 - HI} - p(w, n, dr)$$

$$GY = \frac{p(w, n, dr) - (1 - IC)p(w, n, dr)}{1 - HI}$$

$$GY = \frac{p(w, n, dr) * HI}{1 - HI}$$
(Eq. 23)
$$GY = \frac{p(w, n, dr) * HI}{1 - HI}$$
(Eq. 24)

In which:

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BY – Biomass yield (kg ha<sup>-1</sup>);
GY GY - Grain yield (kg ha<sup>-1</sup>);
SY SY - p(w, n, dr) = Straw yield (kg ha<sup>-1</sup>);
w - 45\% of the rainfall (mm);
n – Nitrogen dose (kg ha<sup>-1</sup>);
dr – Growth regulator dose (mL ha<sup>-1</sup>);
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HI – harvest index obtained by regression equations.

Data were subjected to analysis of variance to detect the main and interaction effects (not shown) and linear and quadratic regression analysis with the t test in the parameters of each equation. The comparison of the results simulated by the hybrid model with those obtained in the field was made using the absolute error and confidence interval of the mean in each observed and simulated resulted in dose of nitrogen and growth regulator. Statistical analyses were performed with the aid of the GENES software.

#### RESULTS AND DISCUSSION

In 2015, at the time of nitrogen application, the soil presented adequate humidity conditions due to the accumulation of rain from the previous days (Figure 1). The high volume of rainfall during the cycle provided periods of less insolation, which reduces the efficiency of photosynthesis. The maximum temperature close to the application of nitrogen was the lowest in relation to the other years. These facts, combined with the average grain yield (Table 1), justify a reasonable yield, characterizing the year as intermediate (IY) to the growing. In 2014, the application of nitrogen was followed by a rainfall volume greater than 50 mm, a volume also observed close to the grain harvest stage. In the application of fertilizer, the average maximum temperature was shown to be the highest in relation to the years 2013 and 2015. These facts justify the lower yield obtained (Table 1), either due to the loss of nutrients due to volatilization or leaching or due to excessive rain occurred at maturation, characterizing the year as unfavorable (UY) for this crop. In 2013, the maximum temperature obtained when nitrogen was applied was around 20°C and there were favorable soil moisture conditions (Figure 1). Under these conditions, although the total rainfall volume was the

lowest, the adequate distribution of rainfall throughout the cycle (Figure 1) was decisive for the highest grain yield (Table 1), above 4 t ha<sup>-1</sup>, characterizing the year as favorable for growth (FY).

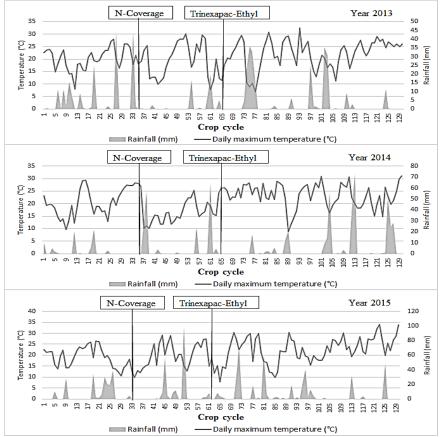


Figure 1- Rainfall and maximum temperature during the oat growing cycle and the timing of nitrogen application and trinexapac-ethyl growth regulator.

For Storck et al. (2014) and Arenhardt et al. (2017) the distribution and volume of rainfall have a significant influence on grain yield. Even in wheat and oats, the conditions of the year are defined by the distribution and volume of rainfall (Hawerroth et al., 2015; Mamann et al., 2017). This fact occurs due to the compromised efficiency of nitrogen absorption, aimed at the elaboration of yield components (Mantai et al., 2016; Costa et al., 2018). In addition, excessive rainfall in the grain filling phase contributes to plant lodging and reduced quality, resulting in yield losses (Kaspary et al., 2015; Krysczun et al., 2017).

To define the adjusted dose of growth regulator to reduce lodging without prejudice to grain yield, the dose of 495 mL ha<sup>-1</sup> was considered, according to data presented in Table 2. This dose weighs the forecast of lodging of plants below 5%, thus making it possible to increase nitrogen doses to increase yield.

**Table 1.** Temperature and rainfall data in the years of oat plant growth and average yield of biomass and grains and classification of the agricultural year quality.

Month	Temperature			Rair	Rainfall		DV	Class
Month	Minimum	Maximum	Average	Average*	Actual	$-GY_{\bar{x}}$	$m{BY}_{ar{\mathbf{x}}}$	Class
			20	)15				
May	10.5	22.7	16.6	149	100			
June	07.9	18.4	13.1	162	191			
July	08.3	19.2	13.7	135	200			
August	09.3	20.4	14.8	138	223	3596b	8835b	IY
September	09.5	23.7	16.6	167	046			
October	12.2	25.1	18.6	156	211			
Total	-	-	-	909	973	='		
			20	)14				
May	11.1	24.5	17.8	149	020			
June	09.3	19.7	14.5	162	059			
July	07.4	17.5	12.4	135	176			
August	12.9	23.4	18.1	138	061	3028c	8076c	UY
September	12.0	23.0	17.5	167	194			
October	15.0	25.5	20.2	156	286			
Total	-	-	-	909	798	='		
			20	)13				
May	10.0	22.6	16.3	149	108			
June	08.9	20.0	14.5	162	086			
July	07.0	20.6	13.8	135	097			
August	06.6	19.8	13.2	138	163	4354a	9768a	FY
September	09.6	21.0	15.3	167	119			
October	13.2	27.1	20.2	156	138			
Total	-	-	-	909	712	_		

<sup>\*=</sup> Historical average of rainfall obtained from May to October 1990 to 2015; Averages followed by the same letter in the column do not differ in the probability of 5% error by the Scott-Knott test; FY = favorable year; UY = unfavorable year; IY = intermediate year; Temperature (°C); Rainfall (mm);  $GY_{\bar{x}}$ = grain yield (kg ha<sup>-1</sup>);  $BY_{\bar{x}}$ = biomass yield (kg ha<sup>-1</sup>).

**Table 2.** Estimation of the adjusted dose of growth regulator to attain oat lodging probability of less than 5%.

N Dose (kg ha <sup>-1</sup> )	EQUATION $LO = a \pm bx$	R²	P(bx)	LOE (%)	Adjusted Dose (mL ha <sup>-1</sup> )
	(	2013+2014+2015)			
30	25.23 - 0.044x	87	*	(5)	<b>≅</b> 460
90	50.53 - 0.090x	87	*	(5)	<b>≅</b> 500
150	76.25 - 0.136x	92	*	(5)	<b>≅</b> 520
30-150	50.67 - 0.092x	89	*	(5)	<b>≅</b> 495

N Dose = nitrogen dose; LO = lodging;  $R^2$  = coefficient of determination;  $P_{(bx)}$  = parameter that measures the slope of the line; () = consideration of the possibility of lodging plants of 5%;  $LO_E$  = estimated lodging; Adjusted dose = regulator dose that allows lodging of plants below 5%; \* = significant at 5% probability of error, respectively, by the F statistic.

Table 3 shows the model that describes the behavior of the harvest index. These results enable a qualified estimate of the biomass and stem ratio as a function of the use of

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growth regulator in each nitrogen dose. The value of 0.40 was considered, as shown in Table 3, this value being independent of the condition of nitrogen fertilization and the dose of growth regulator applied; the result that will be used in the hybrid model.

**Table 3.** Regression equations to estimate the oat harvest index as a function of growth regulator doses, under varying nitrogen use conditions.

N Dose (kg ha <sup>-1</sup> )	EQUATION $IC = a \pm bx \pm cx^{2}$	R²	P(bx²)	Adjusted Dose (mL ha <sup>-1</sup> )	HIE (kg ha <sup>-1</sup> )
·	(2013+2	2014+2015)			
30	$0.35 + 0.00049x - 0.00000069x^{2}$	98	*	460	0.43
90	$0.36 + 0.00015x - 0.00000026x^{2}$	97	*	500	0.37
150	$0.35 + 0.00032x - 0.00000034x^{2}$	95	*	520	0.42
30-150	$0.35 + 0.00032x - 0.00000043x^{2}$	97	*	495	0.40

N Dose = nitrogen dose; HI = harvest index;  $R^2$  = coefficient of determination;  $P(b_x^2)$  = parameter that measures the slope of the line by the probability of T at 5% error; Adjusted Dose = regulator dose for less than 5% lodging predictability;  $HI_E$  = simulated harvest index; \* = Significant at 5% probability of error, respectively, by the F test.

The development of yield simulation models depending on the condition of the agricultural year does not include efficient forecasting models, given the strong variation during each growth year (Figure 1 and Table 1). Therefore, for the purpose of comparison, elaboration and validation of the proposed models, the cumulative effects of the variability existing between the years of growing were considered. Thus, Table 4 presents the results of simulation of white oat straw yield using the IPM-Carvalho model. For the simulations, the variable w was considered 45% of the average rainfall obtained during the oat cycle in the three years of study (370 mm). It is verified that in all nitrogen doses tested, the values simulated by the model were close to the values observed in the field, validating the use of the IPM-Carvalho model to simulate the straw yield in white oats.

**Table 4.** Simulation of white oat straw yield by the internal point method (IPM-Carvalho) based on use of N-fertilizer.

N Dose	IPM-Carvalho Method*	SYE	SYO
	(2013+2014+2015)		
30		5980	5945
90	$35.75w + 15.54n - 0.056w^2 - 0.051n^2$	6545	6530
150		6745	6710

N Dose = nitrogen dose (kg ha<sup>-1</sup>); W = 45% of the average rainfall occurred during the oat cycle in the three years of study (370 mm); N = 100 nitrogen dose (kg ha<sup>-1</sup>); N = 100 estimated straw yield (kg ha<sup>-1</sup>); N = 100 equation 12.

There are models developed for one crop and that are calibrated and validated for others. An example is the work of Mantai et al. (2017) that simulated the white oat development cycle using the WE-Streck model, developed for simulating the wheat development cycle. Freitas et al. (2004) simulated corn yield using the CERES-Maize model as a function of the water depth and application uniformity. Silva et al., (2012) using the CERES-Wheat model simulated the growth and development of wheat in the Campinas region. While Farid et al. (2015) implemented and validated the CERES-Wheat model for simulating application rates of fertilizers in wheat.

Table 5 shows the values simulated by the implemented IPM-Carvalho model, presented in (15), and the observed averages of white oat straw yield. It is noticed that the increase in growth regulator doses reduces the yield of oat straw, justifying the need to include the adjusted growth regulator dose in the model (Table 2). In all tested nitrogen and growth regulator doses, the model presented simulated values close to the real values observed in the field and within the confidence interval of the established average. Analyzing the general model, which considers the use of the adjusted dose of growth regulator on the lodging of plants (Table 2), regardless of the nitrogen fertilization condition, the simulated straw yield value was approximately 5775 kg ha<sup>-1</sup>, close to the value observed in the field of 5725 kg ha<sup>-1</sup> and within the confidence interval established by the average of the different years. Therefore, validating the proposed use of the IPM-Carvalho model implemented to simulate white oat straw yield, regardless of the condition of nitrogen and growth regulator use.

**Table 5.** Simulation of white oat straw yield using the IPM-Carvalho model and averages observed in different crop years.

N Dose (kg ha <sup>-1</sup> )	R Dose	Straw yield (kg ha-1)*		AE	Confidence limits	
	(mL ha <sup>-1</sup> )	Simulated	Observed	(kg ha <sup>-1</sup> )	LL	UL
		(2013	3+2014+2015)**			
	0	5980	5945	35	4913	6581
20	200	5835	5750	85	4739	6385
30	400	5545	5490	55	4514	5850
	600	5110	5020	90	4327	5758
	0	6545	6530	15	5565	7315
90	200	6400	6345	55	5439	7164
90	400	6110	6075	35	4486	6562
	600	5675	5580	95	4267	6451
	0	6745	6710	35	5594	7684
150	200	6600	6635	35	5356	7145
	400	6310	6240	70	4932	6842
	600	5875	5880	5	4155	6634
30-150	495	5775	5725	50	4231	6548

N Dose = nitrogen dose; R dose = growth regulator dose; Simulated = value simulated by the model; Observed = average of the values observed in the field during the three years of study; AE = absolute error; LL = lower limit; UP = Upper limit.; \* = straw yield obtained by equation 15; \*\* = result of 3 years of cultivation.

The calibration and implementation of models for efficient simulation of the yield of a given crop can be a highly effective tool and can assist in making decisions regarding agricultural investment. Silva et al. (2018) implemented the AQUACROP model to simulate the soybean crop grown under different levels of irrigation. Walter et al. (2012) implementing the INFOCROP model efficiently simulated the grain yield of the irrigated rice crop. Lima Filho et al., (2013) successfully implemented the CROPGRO model to simulate the grain yield of cowpea in the Recôncavo Baiano. Costa et al., (2014) implemented the APSIM-Sugar model based on biological and environmental indicators, simulating sugarcane yield with high quality.

Table 6 presents the biomass yield values simulated by the model proposed in (20) and the average of the real values obtained during the three years of study. For these simulations, the value of 0.40 was considered for the harvest index (HI), value

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obtained from the results presented in Table 3. In all conditions of nitrogen fertilization and use of growth regulator, the model for estimating total biomass showed satisfactory behavior, with simulated values close to the observed values and in the mean confidence interval. Even in the general model, regardless of the condition of nitrogen fertilization and considering the adjusted dose of growth regulator to the lodging of plants (Table 2), they obtained a simulated value of 9625 kg ha<sup>-1</sup> of biomass, a value close to the real observed in the field of 9750 kg ha<sup>-1</sup>.

**Table 6.** Comparison between observed and simulated values of white oat biomass yield with different doses of nitrogen and growth regulator.

N Dose (kg ha <sup>-1</sup> )	R Dose (mL ha <sup>-1</sup> )	Biomass Yield (kg ha <sup>-1</sup> )*		AE (kg ha <sup>-1</sup> )	Confidence limits	
		Simulated	Observed		LL	UP
			(2013+2014+2015)**			
	0	9970	9845	125	9213	12581
20	200	9725	9450	275	8839	11385
30	400	9240	9095	145	8314	9650
	600	8520	8480	40	7755	9158
	0	10905	10700	205	9174	11427
00	200	10670	10450	220	9080	11230
90	400	10180	10125	55	8820	11310
	600	9460	9385	75	8561	9888
	0	11240	11100	140	9695	12965
1.50	200	11000	10880	120	9558	12286
150	400	10515	10325	190	8830	11763
	600	9790	9640	150	8417	10583
30-150	495	9625	9750	125	8560	10312

N Dose = nitrogen dose; R dose = growth regulator dose; Simulated = value simulated by the model; Observed = average of the values observed in the field during the three years of study; AE = absolute error; LL = lower limit; UL = Upper limit; \* = biomass yield obtained by equation 20; \*\* = result of 3 years of cultivation.

Table 7 shows the yield values of white oat grains simulated by the model proposed in (24) and the average of the real values obtained in the field during the three years of study. For these simulations, the value of 0.40 of the harvest index was considered, the same used previously to estimate the biomass yield. In all conditions of nitrogen fertilization, the simulation of white oat grain yields presents values close to the average of the values observed in the field during the three years of study. In addition, as already presented, the increase in the growth regulator doses provides a linear reduction in the yield of white oat grains, thus, the results presented by the proposed model also show this same behavior trend. In a general analysis, using a dose of growth regulator for lodging of plants of maximum 10%, with technical indication of nitrogen fertilization of 70 kg ha<sup>-1</sup>, for an estimate of grain yield around 4000 kg ha<sup>-1</sup>, the proposed model presents simulated yield value (3850 kg ha<sup>-1</sup>) similar to the results observed in the field (4025 kg ha<sup>-1</sup>) and in the confidence interval of the established average. Thus, the results presented validate the use of the IPM-Carvalho model combined with the polynomial equation for simulating the yield of white oat grains, regardless of the condition of nitrogen fertilization and dose of growth regulator.

**Table 7.** Comparison between observed and simulated values of grain yield of oats with different doses of nitrogen and growth regulator.

N Dose	R Dose	Grain yield (kg ha <sup>-1</sup> )* Simulated Observed		Absolute	Confidence limits	
(kg ha <sup>-1</sup> )	(mL ha <sup>-1</sup> )			error	LL	UL
		(2013+	2014+2015)**			
	0	3990	3900	90	3789	4429
20	200	3890	3700	190	3584	4273
30	400	3695	3605	90	3401	4080
	600	3405	3460	55	3115	3888
	0	4360	4170	190	3769	4587
00	200	4265	4105	160	3664	4427
90	400	4075	4050	25	3565	4348
	600	3785	3805	20	3421	4198
	0	4495	4390	105	4095	4758
1.50	200	4400	4245	155	3962	4655
150	400	4210	4085	125	3747	4582
	600	3915	3760	155	3402	4144
30-150	495	3850	4025	175	3556	4384

N Dose = nitrogen dose; R dose = growth regulator dose; Simulated = value simulated by the model; Observed = average of the values observed in the field during the three years of study; AE = absolute error; LL = lower limit; UL = Upper limit; \* = grain yield obtained by equation 24; \*\* = result of 3 years of cultivation.

Models that represent reality through one or more equations and algorithms, enable efficient and low-cost simulations (Corrêa et al., 2011; Mamann et al., 2017). This need becomes more necessary for cultivated agroecosystems, as they represent real situations of non-linear behavior and act directly on yield. Therefore, models that simulate the yield of agricultural crops are increasingly sought after, contributing to the realization of studies and validation of more efficient and sustainable managements (Costa et al., 2016; Marolli et al., 2017c; Piekarski et al., 2017). In this perspective, Carvalho et al. (2009) developed a model of internal parameters (IPM) for the calculation of dry oat matter, such model considers p(w,n) to be the response function (t ha<sup>-1</sup>) in relation to the water layer w (mm) and the nitrogen dose (n) applied in topdressing (kg ha<sup>-1</sup>). In general, the model is characterized by being a non-linear function, with lower and upper bounds of w and n. It is noteworthy that for the calculations of dry matter in white oats in the IPM-Carvalho model, the values of water availability of the soil were considered, being it equal to 45% of the rainfall, with the w value being adopted of 280 ml for the crop total cycle. Computational limitations until a few years ago meant that models with biological significance and statistical analyzes proved to be conflicting, leading to a preference for linear and/or polynomial models in relation to non-linear models, with greater biological significance. The computational advances already allow the use of nonlinear models of phenomena aggregating statistics and modeling together with biological processes, as well as the correct use of analysis of repeated measures in time, generating greater reliability for direct application in computer programs and applications in agriculture devices (Mamann, et al., 2019; Trautmann, et al., 2020).

## **CONCLUSIONS**

The IPM-Carvalho model can be used to simulate white oat straw yield. Its implementation along with regression models enables simulation of biomass and grain

yield, based on nitrogen fertilizer and growth regulator doses, ensuring efficient predictability.

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# **CONFLICTS OF INTEREST**

The authors declare no conflict of interest.

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